REVIEW OF THE GRAVITY MODEL: A PHYSICAL APPROACH TO SOCIAL SCIENCES

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Abstract. *This article presents a bibliographic review of the gravitational model in sociophysics since its origins in Newton's conception. Considering the crucial role played by physics in the expansion of our understanding about social sciences, I will preserve such an approach throughout the article. Firstly, I will introduce its theoretical roots with an overview of the physical models by the most prominent scholars, up to the recent applications in migration and economics. Secondly, I will illustrate the analogies and differences between the gravitational model and Tobler's first law of geography. Thus, this work results in a sound comparison between a simple spatial framework in three variables, the most notable of which is distance, and a structured scheme based on the gravity metaphor which relies on the concepts of attractions' forces and spatial concentration. Thirdly, I will propose an assessment of the most recent application to intangible goods, as in the fields of marketing and services.*

Keywords: *gravity model, demography, migration, sociophysics, economic geography, social sciences*

1. INTRODUCTION

In physics one of the universal forces regulating the entire universe and acting as a fundamental condition of all our movements is gravity. Such concept has also been successfully applied to social sciences, under the name of 'gravity model'. This article, therefore, is intended to be a historical review of the origins and applications of the gravitational model to social sciences, with a focus on its roots in physics. Whilst it should be mentioned that there is a similar review of thirty years old academic literature on the topic (Philbrick, 1973), the present study represents a step forward, for it offers an updated and detailed analysis with a broader range of the existing applications of the model.

Whilst filling a void in the academic panorama, the study is suggesting an updated thematic division of the gravity model applications to social sciences which can clarify its socio-physical evolution.

This article starts by examining the principles of the gravitational model, from Isaac Newton's formula to its recent applications in the broader field of social sciences, including international trade. In order to respect the mathematical and physical origins of this model, the initial approach rooted in physics and mathematics will be preserved throughout the bibliographic review.

In the consecutive pages, an attempt will be made to answer the following key question: what are the main features of the developments of the gravity model in social sciences? The structure of the review is organised in two paragraphs. The former aims at introducing the main pillars of gravity in physics which are necessary to understand the application of gravity to social sciences, given the physics-based origins of the model. The latter ranges from the theoretical foundations of this application to the review of the different uses of gravity in social sciences, such as in the study of people's movements. In this sense, the Tobler's Law becomes an indispensable point of reference to grasp the theoretical evolution of the gravity model[1].

As aforementioned, this article introduces an innovative approach because its purpose is to trace the theoretical applications of the gravity model over time and to analyse in detail its contribution to social sciences with the aim of achieving a deeper understanding of human behaviour and social phenomena. Moreover, the element of novelty consists in the suitable combination of the sequential scheme with a thematic division into subparagraphs.

I will follow a chronological order, which is the most appropriate to describe the interdisciplinary development of the application of the physical model. Against this background, three historical evolutions emerged with respect to the applications of gravity to social sciences:

- Application to people's movements.
- The shift from people's movements to the interest in flows of things (economic flows); and
- A recent interest in less tangible flows (ideas, services, et cetera).

[1] For example, it would be difficult to fully understand Isard's gravity model without the prior study of Stewart's.

1.1 Isaac Newton's Law

The gravity model owes its name to the homonymous law in physics.[2] The discovery of gravity law is not due to an individual work but it is the result of the contributions of several scientists.[3] As famously stated by Newton himself: "If I have seen further, it is by standing on the shoulders of Giants" (Newton 1675). The Earth exerts a force of attraction towards the ground (as in the case of the apple falling from the tree). Newton realized that this rule was not only valid on the surface of the Earth but was equally applicable throughout the universe (universal law), being directly related to the mass of thing, of both terrestrial and celestial bodies. Johannes Kepler (1609) had previously described how planets move through space following elliptical orbits.[4] In addition to demonstrating the accuracy of this model, Newton further developed Kepler's Third Law of planetary motion, which described how planets move in ellipses with the sun always at one focus of the ellipse. Therefore, for instance, the same force was responsible for keeping the Moon in Earth's orbit. This law is defined in physics with the famous formula of the Law of Universal Gravitation proposed by Newton (1687). With respect to the terminology used, the word 'gravity' derives from Latin gravitas, meaning heaviness (weight); its first appearance in a scientific text, written in Latin, dates back to the book Philosophiae Naturalis Principia Mathematica, published in 1685 (Schmitz 2018: 251). This law is universal, and it postulates an invisible force of attraction of matter which acts through both long and short distances. Indeed, in the universe two particles of matter attract each other with a force directly proportional to the product of the masses and inversely proportional to the squares of the distance between them. The closer the bodies are and the bigger their masses, the higher their attraction is to each other. In the formulation of this law, Newton created the concept of mass, that is, the quantity of matter of a body (volume by density), which was distinguished from that of weight, which varies according to the acceleration of gravity. The measure of weight changes in space, while mass is a fixed quantity which does not vary. The larger the mass of a body, the more powerful the force of gravity. The formula developed by Newton is the following (Schutz 2003: 13-18):

$$F_{ij} = G\ \frac{M_i M_j}{D_{ij}^2} \qquad (1)$$

F_{ij} is the attractive force, M_i and M_j are the masses, D_{ij} is the distance between the centres of the two objects, G is a gravitational constant (universal gravitational constant[5]) depending on the unit of measurement of mass and force, the subscripts i and j indicate the pair of bodies considered.[6] The force will have the same direction as the intersection line joining both material points; bodies with spherical mass distribution attract and are attracted as if all their mass was located in their centres.[7]

Since gravitational attraction is expressed as a force in physics, its vector form is important. Indeed, force manifests itself in the mutual interaction of two or more bodies, which vary their state of rest or uniform motion, it is expressed as a vector quantity because it does depend on which direction that force is applied.[8] Considering its effect on one of the bodies, its vector will be composed of a magnitude (also called length or norm), a directed axis and a direction (also called versor, unit vector or orientation). The applied intensity will be its magnitude, which is exerted between two points in space (directed axis) and from one point to another (direction).[9] In the presence of multiple bodies and

[2] Unless otherwise specified, all the contents presented are based on the following textbooks, as indicated among the bibliographic resources: James B. Hartle, Gravity: an introduction to Einstein's general relativity (London: Pearson, 2003); Bernard Schutz, Gravity from the ground up - An Introductory Guide to Gravity and General Relativity (Cambridge: Cambridge University Press, 2003), 9; Charles W. Misner, Kip S. Thorne, and John A. Wheeler, Gravitation (Freeman, 2000).
The reader unfamiliar with this field of studies is referred to these books, or to any other introductory textbook, for a better explanation of the concepts and formulations used in this paragraph.

3 Among these, Kepler (1571-1630), Galileo (1564-1642), Descartes (1596-1650), Borelli (1608-1679), Huygens (1629-1695), Robert Hooke (1635-1703). Their studies laid the foundation of Newton's law of universal gravitation contained in the Principia (Jourdain 1913:353-384).

4 Kepler explained the motion of celestial bodies with its three laws. The first law stated that the planets describe around the sun elliptical orbits where the sun represented one of the focal points. The second law held that the motion of the planets was uniform. The third law stated that the squares of the planets' motion times to describe the orbits were proportional to the cube of the length of the same major axis of its orbit.

5 It is called also Big G for disambiguation with the local gravitational field of the Earth (small g). Newton did not know the precise value of G but tried to approximate it. Its value is difficult to measure because it is weak as compared to the other force, but it acquires relevance for celestial bodies' masses. It is approximately 6,67 x 10-11 N m2 kg -2 (that is newton per square meter divided by square kilogram).
Being small g the acceleration, equal to 9,8 m/s2 on the Earth (that is meter divided by square second), we will have that, for the object i, g= (GM_j)⁄(D_ij^2).

6 Force is measured in newton (N), masses are measured in grams (g), and distance in meters (m).

7 The Shell Theorem formulated by Newton in Principia, probably inspired by Robert Hooke's theories of centripetal gravity of celestial bodies, states that a spherically symmetric mass, having uniform density, exerts a force on the outside as though all its mass was concentrated at its center, while exerting a zero force inside.

8 In physics, we distinguish between scalar quantity and vector quantity. The former is a quantity described solely by a real number (magnitude) that does not depend on the direction, while the latter has a magnitude and a direction (which technically is the ensemble of directed axis and orientation). Common vector quantities are force, velocity and acceleration. Common scalar quantities are distance, speed and energy.

9 A vector, metaphorically like an arrow, with a certain intensity (magnitude) will follow a trajectory along its rod (directed axis)

their respective attraction forces, the force deriving at a point will be the result of the vector composition of the attraction forces considered (vector sum[10]). The vector form is the same of the equation (1), except that F is now a vector and the right-hand side is multiplied by the unit vector from object i to object j.

$$\vec{F}_{ij} = -\,G\ \frac{M_i M_j}{|\vec{D}_{ij}|^2}\ \frac{\vec{D}_{ij}}{|\vec{D}_{ij}|} \qquad (2)$$

$\vec{F}_{ij}$ is the attractive force applied to object j and exerted by object i, M_i and M_j are the masses, $|D_{ij}| = |D_j - D_i|$ is the distance between the centres of the two objects, G is a constant gravitational (universal gravitational constant), $\frac{\vec{D}_{ij}}{|\vec{D}_{ij}|} = \frac{D_j - D_i}{|D_j - D_i|}$ is the unit vector from object i to object j. In the vector form, we will have that $F_{ij} = -F_{ji}$.

In ancient times, it was thought that forces originated exclusively because of contact.[11] Instead, according to the law of universal gravitation, two objects can interact with each other through instantaneous action at a distance. Newton observed the phenomenon of gravitational attraction in various experiments and demonstrated it mathematically, without knowing its cause (hence his famous quote "hypotheses non fingo").[12] Under the pressure of the 19th century's discoveries by Faraday (Ulaby and Ravaioli 2007: 255) and Maxwell in the context of Electromagnetism,[13] the concept of a field was introduced, which, unlike the Newtonian idea of instantaneous interaction between bodies, identified a set of values that a physical quantity assumed in a region of the space. Ergo, space ceased to be purely geometric: it expressed quantity as a function of position. In this sense, gravity can be described through a vector field in which the magnitude that varies is the force of gravity. This field can be represented graphically through an aura of force lines that from every point in space are directed towards the generating mass' centre and that will be more intense near the surface of the mass. In this regard, the gravitational field equation is defined as the ratio of the force of gravity $\vec{F}$ acting on the mass m, where the gravitational field vector $\vec{g}$ is measured as an acceleration (m/s^2) and is parallel and with the same direction/orientation of the force[14].

$$\vec{g} = \frac{\vec{F}}{m} \qquad (3)$$

Considering the intensity of the gravitational field in a single point with distance r from the generating mass' centre of the field, as in the case of the Earth exerting a gravitational field on a mass located at that point, there will be an oriented force towards the centre of the earth equal to:

$$\vec{g} = \frac{G\,M}{D^2} \qquad (4)$$

The intensity of the field will depend on the size of the mass generating it and the distance (D).[15]

The energy associated with the mutual influence of two gravitational fields interacting with each other is called Gravitational Potential Energy, namely the work against gravity performed in moving the mass of an object to a given point in the gravitational field of another body and it is ready to be transformed into another form of energy.[16] Its formula is:

$$U = -\,G\,\frac{M\,m}{D} \qquad (5)$$

where U is the gravitational potential energy[17], G is the gravitation constant, M is the mass of the attracting body, m is the mass of the attracted body, and D is the distance between their centres. More commonly, M represents the mass of the earth. The negative sign indicates the energy expenditure of the body m moving in the attracted direction.
Considering m as a point unit[18] we will get the gravitational potential (V), which does not depend

proceeding from the arrow's nock towards its tip (direction or versor).

10 The interested reader will be able to further explore the rules of vector sum in any elementary manual of physics or engineering. I suggest Bruce R. Kusse and Erik A. Westwig, Mathematical Physics: Applied Mathematics for Scientists and Engineers, Second Edition (Weinheim: WILEY-VCH, 2006), 1-17 and 44-58.

11 That is in physics a push, a traction or a shock.

12 For Newton, it is not possible to speculate on something that cannot be proven. In Principia, we read: "I have not as yet been able to discover the reason for these properties of gravity". For a rational explanation of the cause of gravity, we have to wait until 1915, when Albert Einstein presented the Theory of general relativity to the Berlin-Brandenburg Academy of Sciences and Humanities.

13 It is worth noting that Maxwell's equations for electromagnetism have been called the second great unification in physics, where the first one was realized by Isaac Newton (Feynman et al. 2011, cap.18) and the third will be realized by Einstein.

14 See supra note 10.

15 In the terrestrial case for r equal to the Earth's radius, it results that the acceleration of gravity on the Earth's surface is equal to 9.8 m/s2.

16 More specifically, potential energy is stored energy. The body of an object is endowed with it due to its position or orientation.

17 Another more general form considers the distance from the earth (h) and the Earth's gravitational acceleration ($g = 9.8$ m/s^2). Its formula is $U = gmh$.

18 Inversely, considering M as a point unit we will obtain the gravitational potential (V) that does not depend on the possible mass quantity placed in it. The gravitational potential which the mass m produces at that point of the attractive mass (V_M) will be therefore $-\,G\frac{m}{D}$.

on the potential quantity of mass placed in it. The gravitational potential produced by the mass M at that point of the attractive mass[19] will therefore be equal to:

$$V_m = -G\frac{M}{D} \quad (6)$$

2. THE CONCEPT OF GRAVITY IN SOCIAL SCIENCES

2.1 Theoretical Foundations

Among the earliest applications of science to social phenomena[20] the study by the astronomer Edmond Halley (1693) should be recalled.[21] He calculated the "just value to be paid for an Annuity during the whole term of the Lives" combining demographic concepts with the calculation of probabilities for risk assessment.[22] However, the concept of social physics as such only emerged in the 19th century, starting with the Lettres d'un habitant de Genève à ses contemporains (1803) by the French philosopher and economist Saint-Simone. This work was a tribute to science, provocatively seen as a new religion devoted to Newton's cult and aimed at the union of the most important scholars in the world.[23] The French philosopher aspired to the integration of the scientific principles of different fields of study through a revolution of Western and European thought, which had to put science at its centre. Thus, the scholar had the idea of describing society using laws such as those of physics and biology. However, it was his pupil Auguste Comte (1854: 39) who first used the term social physics, defining it as "that science which occupies itself with social phenomena, considered in the same light as astronomical, physical, chemical, and physiological phenomena, that is to say as being subject to natural and invariable laws, the discovery of which is the special object of its researches". Later, the Belgian astronomer Adolphe Quetelet (1835) resumed the term social physics in his work A Treatise on Man and the Development of his Faculties[24], where he analysed society through statistical probability[25], comparing the individually measured observations with their normal distribution, which was obtained from a sample. Indeed, through the application of the scientific method to the analysis of human phenomena the astronomer outlined what today we would call the social sciences. When Quetelet adopted the term social physics, Comte coined the word sociology to circumscribe his field of study[26], perhaps for reasons of rivalry (Jahoda 2015).

However, over time social physics acquired the meaning of a more specific field of study aimed at the application of physics to the social sciences, while according to Comte, sociology referred to the science collecting all studies on humans (such as history, economics, psychology, et cetera).[27] Indeed, social physics (also called sociophysics) is defined today as an interdisciplinary field of study that uses mathematical tools developed in physics to deepen our understanding of social phenomena and human behaviours. Among these, the concept of gravity has been widely applied to social sciences' studies. Models based on this force of attraction are part of a large category of frameworks, which are described as spatial interaction models.[28] Their formulas are mainly, yet metaphorically, based on Newton's law of universal gravitation (1) or on the gravitational field's equation (4). These functional forms have been applied to a whole series of 'social interactions', such as migrations[29], tourism, trade, foreign direct investment, demographics, cultural exchanges and more. More generally, the application of the gravity model to social sciences predicts a flow between partners (countries, regions, companies, or other subjects) in an intuitive way.[30] Drawing on gravity's laws application to social

[19] When there are more than two masses in a given space, the total potential will be equal to the sum of the bilateral potentials

20 The interested reader will be able to explore more in-depth some scientific applications to the field of social sciences before Halley's ones in: Gianfranco Tusset, From Galileo to modern economics: The Italian Origins of Econophysics (Springer International Publishing, 2018).

[21] Halley's most important discoveries include identification of solar warming as the cause of atmospheric motions, the relationship between barometric pressure and sea level height, the calculation of an elliptical orbit for a visible comet, now known as Halley's comet. Halley was also interested in the gravity and proof of Kepler's laws of planetary motion, for which he financed and supported his colleague and friend Newton. Newton published the book Philosophiae Naturalis Principia Mathematica thanks to Halley's funding.

[22] This provides the theoretical basis for the future birth of the life insurance sector.

[23] This idea will be better explained in 1814 by Saint-Simone in "Réorganisation de la société européenne".

[24] The original title in French is "Sur l'homme et le dévelopment de ses facultés, ou Essai de physique sociale".

[25] The scholar collected statistics on madness, drunkenness, crime and suicides by establishing relationships with some predictive variables.

[26] Comte used the term 'sociology' because he disagreed with Quetelet's ideas that a theory of society could be the result of a collection of statistics.

[27] Although, as already mentioned, the terms social physics, sociology and social sciences were coined in the same period, and at the time they were often used with the same value and meaning, they have acquired over time different nuances. In particular, social sciences are intended in a broader sense that differs from the natural sciences, since the former (social sciences) focus on human activity. Sociology, instead, is a specific social science that studies society and its relationship with the individual.

[28] According to geographer Peter Gould (1958:57), Geographic theory "is a matter of gravity".

Finally, social physics refers to the application of physics and its rules (physics is a natural science) to the social sciences.

[29] See the gravity model of migrations in Section 2.3.

[30] Detractors of the gravitational model accuse it of being only based on observations, in addition to being biased towards the largest centres and grounded on historic ties.

sciences, the aforementioned flow is attracted in proportion to a specific measure of size and proximity, depending on frictions of the distance, mainly expressed in terms of distance, time or cost. According to economist Walter Isard (1954: 308) "the distance variables act in much the same manner with respect to the social world as to the natural world". Therefore, a gravitational relationship must arise in any model that considers distance to be a direct factor that implies costs. Carrothers (1955: 99) argued that the analogy between human interaction and Newtonian physics of matter was possible. Although "it may not be possible to describe the actions and reactions of the individual human in mathematical terms", the behaviour of a set of people was instead predictable thanks to the study of probability in mathematics. According to the author, this phenomenon was observable in all the social sciences, since people behave differently in groups than they do as individuals.

Similarly, in physics it was not possible to describe the behaviour of molecules taken individually, while it was possible to predict the action of a set of molecules. The logical considerations just mentioned, in particular those of Compte (1854), Isard (1954), and Carrothes (1955), represent the necessary premises to develop a physic reasoning in the social sciences, and without which a gravitational sociophysical structure would make no sense, due to the lack of logical coherence/consistency. In a certain sense, one is free to believe or not that human actions are guided by universal laws. Nevertheless, beyond the solution to this complex dilemma, one is obliged to accept a sort of determinism in human beings' life, if one wants to use a gravity model in the social sciences.

In social sciences, the first component considered is the attribute's size[31] (quantitative variable), which is positively related to the phenomena under examination (the formula's result which is usually a flow). On the contrary, the second element, distance, refers to the spatial and geographical dimension: more specifically, it includes all the variables acting as 'resistance factor' that affect negatively the phenomena examined. This law of general gravity for social interaction can be expressed in the following formula notation:

$$F_{ij} = \frac{G\,(M_i^{\alpha}\, M_j^{\beta})}{D_{ij}^{\theta}} \qquad (7)$$

Where F_{ij} is the flow created between i and j or, alternatively, the total volume of the interactions between i and j (that is the sum of the flow in both directions: from i to j and from j to i). It can represent, for example, the value of exports or migration. M_i and M_j are the relevant dimensions of the two subjects i and j (regions, countries, population, enterprises or others), while D_{ij} is the distance[32] between subjects i and j (usually measured from centre to centre or with an estimation of the resistance factor); α, β and θ[33] are parameters that adapt according to the type of social interaction taken into consideration (migration, tourism, foreign direct investment, et cetera). For the flow of people, it is more natural to measure M_i and M_j as populations. Note that we return to Newton's law equation (2) if $\propto = \beta = 1$ and $\theta = 2$.

According to the vector form of the law of gravity in physics, the property distinguishing gravitational models refers to the total flow from a source ($Origin_i$), which is the sum of that flow directed to all the possible destinations. Likewise, the total flow to a point ($Destination_j$) refers to the total flow that is directed from all origins towards that point. In formal terms (Philbrick 1973: 40-41):

$$Origin_i = F_{i1} + F_{i2} + F_{i3} + \cdots + F_{in} = \sum_{j=1}^{n} F_{ij} \qquad (8)$$

$$Destination_j = F_{1j} + F_{2j+} + F_{3j} + \cdots + F_{nj} = \sum_{i=1}^{n} F_{ij} \qquad (9)$$

$Origin_i$ is the total outward flow from i to all j. $Destination_j$ is the total inward flow terminating at a particular destination zone, j, from all origins, i. Gravity models are usually used to estimate the magnitude of the flows (F_{ij}) through the sum of $Origin_i$ and $Destination_j$, or to evaluate the total magnitude for a source or perceived at a point. According to Philbrick (1973)[34], models of the first type can be designated as gravity demand models, while the second as distribution models.

2.2 Pownall's Political Model of Gravity

The British colonial official and politician Thomas Pownall (1764) applied, albeit in a purely theoretical way, the Newtonian concept of gravity and attraction

[31] The size can be measured as gross domestic product, population, product offering, or other variables.

[32] In most of recent studies the distance is not only geographical, but it considers also different elements (morphology, communication infrastructures, cultural factors, etc.).

[33] The square of the distance, which in physics specifies a precise empirical relationship between different measurement systems (for masses, distances and forces), is left apart in some studies in the fields of social sciences. Unlike physics, in these cases the relationship is more theoretical than empirical, since the latter provides index that cannot be measured directly in nature. It is worth noting these two different approaches in the next paragraphs. For example, for Stewart (1941) we will see that the square of distance is not superficial but substantial for his studies in the social sciences. As in physics, in fact, he makes a distinction between his formulas of 'demographic energy' and 'demographic gravitation'.

[34] Philbrick makes this distinction for the 'Transportation Gravity Models' which can however be taken up in a more general sense (Ibid.)

to his theories of empire in order to describe a system of economic and political influence between states and across different social classes.[35] Historically, these studies represent a stand-alone theory, which are remarkable for the breadth of their theoretical scope. Pownall's work anticipates in a universal way many of the successive applications of gravity without, however, limiting its use to a single field of study.

According to the British politician, political power gravitated around property, so that every government in history has always reflected the wealthiest part of the population of that specific time. The gravity of property was exerted between different classes and between different regions in space. The latter was perceived as the area where different gravitational fields in the property interacted with each other. Thus, for example, the Colonies of Great Britain gravitated around the mother country for its economic power.

A change in the structure of ownership between different classes or between different regions would automatically imply a shift in the political system. The gravity centre of power was neither fixed nor permanent but rather mobile, and it followed the economic power. This, at Pownall's time, found its greatest expression in trade. The relationships were based on the interconnection of productive activities, among which work was the gravitational force that held together and regulated society.[36] Against this backdrop, the official had predicted that over time, because of the property's law of natural gravity, the centre of gravity would have shifted from Britain to the colonies of North America, or more generally from European space to the Atlantic areas, which thanks to capital accumulation would have soon gained commercial and economic primacy.

2.3 Application of Gravity to People's Movements

The first systematic and scientific applications of gravity to the social sciences trace back to the 19th century.[37] On top of all social interactions, the study of gravity for immigrant flows was the first to be conceptualized.[38] Metaphorically, as already mentioned, these studies transpose gravity's universal law, the product of the masses and the inverse of the square of distance to human choices (Sen and Smith 1955). With regard to migration and displacement, the gravity model is used in urban geography to estimate traffic flows, migration between two areas and the attraction of people to a centre or to multiple centres. The idea is that more populous and less distant centres have a greater flow of movement between them. Migratory phenomena might or might not be linked to free movement areas of people and migration laws. On the one hand, they are connected since these movement areas and laws often tend to play a decisive role for migratory flows. On the other hand, since migration cannot always be regulated and controlled, movement areas and migration laws are not completely effective. Many elements such as politics, language, culture, taxation and more have an impact on the distance perceived by migrants or even on a country's attractiveness. Such factors are in turn influenced by agreements on free trade and, more generally, on movements of people.[39]

First, Carey (1858: 42) applied Newton's universal law of gravity to "railway traffic and migration" by tracking a tendency for the individual to "gravitate to his fellow man". According to the scholar, the individual, as a component of society ('molecule of society'), is subject to a force of social attraction to other social agents, just as matter in physics is subject to a force of gravitational attraction with other matter. Ravenstein (1885) described this concept in the migration law, according to which a 'populated centre' attracts migrants from other populated centres in relation to the size of the population and inversely related to distance between centres. The mathematical function is as follows:

$$M_{ji} = f\left(\frac{P_i}{D_{ij}}\right) \qquad (10)$$

where M_{ji} is the migration flow from centre j to centre i[40], $f(P_i)$ is a function of population size i and

[35] One of the main objectives was to analyse the geopolitical balances of the British Empire.

[36] The interested reader will be able to deepen in: Matilde Cazzola, "Space as gravitational field: the empire and the Atlantic in the political thought of Thomas Pownall." Global Intellectual History 2(2018): 178-201.

[37] The idea that human behavior varies in a non-linear way, but with the square with respect to a given variable, has been "powerful and durable" over time (Anderson 2004: 334). In the social sciences, we can find the use of the square in different works. For example, Malthus predicts continuous problems of sustainability of the geometric growth of the human population with respect to the arithmetic growth of necessities. (Malthus 1798). In 1890, the statistician and sociologist Francis Galton, father of the regression, realized that for the mathematical use of the square of the deviation "there seems to be a wide field for the application of these methods to social problems" (Galton 1890). However, especially since the use of the law of gravity in the social sciences, the idea of a squared exponent of distance has not always been used. It was rather maintained a gravitational structure that takes into account the attraction force (in general called 'magnetism') of a variable considered and a more generic use of friction or resistance of the distance calibrated on the specific phenomenon considered (Sen and Smith 1955). However, this is not true for all scholars, as we will see for example in Stewart's (1941) model, which gives theoretical importance to the square of distance.

[38] The interested reader can also see the studies by Helliwell in 1997 and Portes and Rey in 2005.

[39] For these reasons, I considered trade agreements as the third macroeconomic lever of the economic power and the agreements on people's movements as the fourth invisible macroeconomic lever. The latter is invisible because these agreements are more difficult to evaluate economically.

[40] This should be clear considering that in the formula it is the population of the country i that attracts the migratory flows.

D_{ij} is the distance between the two population centres.

Contrary to Carey's argument, Young (1924), whose migration formula is based on the gravity law of physics, states that human behaviour "does not lend itself to exact mathematical formulation". This approach is similar to the one used by Ravenstein:

$$M = k\left(\frac{F}{D^2}\right) \tag{11}$$

where M is the estimate of migration between two communities, k is a constant, F is the intensity of attraction and D is the distance between the two communities. Therefore, mutual migration will be directly proportional to the intensity of attraction and inversely proportional to the square of the distance between the communities.

In subsequent years, Reilly (1929) extended the concept of social gravitation to consumers and cities.

The author describes how the city's force of attraction exerted on consumers in the surroundings is directly proportional to the size of the population, and inversely proportional to the square of the distance between the two.[41] Therefore, ceteris paribus, consumers will be more attracted by larger cities rather than by smaller ones. Nonetheless, in accordance with Newton's law, the level of attractiveness would be neutralized at the specific 'breaking point', where consumers will be indifferent to either of the two competing cities, as demonstrated in the following formula (Reilly 1931):

$$\frac{P_i}{D_i^2} = \frac{P_j}{D_j^2} \tag{12}$$

where P_i and P_j are, respectively, the size of the centre i and the centre j, D_i is the distance of the indifference point from the centre i and D_j is its distance from the centre j.

The astrophysicist John Q. Stewart (1941) applied the laws of physics to the study of social sciences, thereby creating the theoretical basis for subsequent studies in social physics. Through his studies of the empirical regularities associated with distance in the social sciences, the scholar found that the law of gravity could also be used to describe demographic phenomena.

To that end, the author adopts the concept of demographic gravitation, according to which a large number of people (for example the population of a big city) acts as force of attraction for other individuals who eventually choose to migrate there. The formula is as follows:

$$F_{ij} = k\left(\frac{P_i P_j}{D_{ij}^2}\right) \tag{13}$$

where F_{ij} is the interaction force between the demographic centres i and j, k is a constant[42], P_i is the population of the area i, P_j is the population of area j, and D_{ij} is the distance between i and j.[43] More specifically, Stewart proposed to adopt different values for the population by nationality, considering the 'molecular weight' of each population's member. With the aim of standardizing the measures, the author chose the molecular mass of the American average as a unit. The latter, for example, will be distinguished from an Australian aborigine, whose molecular weight will presumably be less than one.

The theory further evolved with the concept of demographic energy and potential of population by Stewart (1948). The author measured on the one hand the mutual energy between the populations within the gravitational field and on the other hand the intensity of the magnetism of a demographic centre. The astrophysicist's hypothesis was that demographic energy can be interpreted as the number of human relations per unit of time.[44] Stewart made assumptions of applicability while leaving space for further hypotheses and possible developments in the future.

The content and method of his approach was based on the suggestive interdisciplinary formulas rooted in physics, that in the author' s view, can be successfully applied to the social sciences. He was rather convinced that human phenomena could be studied with the same rigour and through the same laws of physics that regulate nature. Hence, the scholar developed hypotheses of application, which would be intrinsically coherent with the rules of physics. Since Stewart's formulas are entrenched in physics, this approach should never be abandoned. Indeed, the future application of such equations to other socio- and econophysics fields of research should maintain its physical foundation to avoid the

[41] The distance considered by this law is a straight line: obstacles such as mountains, rivers, deserts, etc. are not taken into account.

[42] Stewart postpones the determination of G to the future, with the suggestion of simplifying the units of measurement of population and distance to reduce it to unity.

[43] Like Reilley, Stewart considers straight line distances, while providing several examples where distance should be otherwise interpreted.

[44] Stewart (1948: 56) compares human relations to an "impulse which happens". Moreover, the author adds that the accumulating total of such happenings is the integral of energy with respect to the time, a concept that is similar to 'action' in physics.

risk of conceptual mistakes.[45] Starting from the concept of demographic gravitation, he advanced in demographic analysis following the same laws that are valid in physics. In addition, he used the concept of demographic gradient which simply measures the concentration of people per square mile. In formal terms[46]:

$$E_{ij} = k\left(\frac{P_i P_j}{D_{ij}}\right) \quad (14)$$

$$V_{ij} = K\left(\frac{P_i}{D_{ij}}\right)$$

$$gradient = K\left(\frac{P_i}{m^2}\right)$$

being E_{ij} the demographic energy of the two populations, V_{ij} is the population's potential for attraction to centre i, K is a constant, P_i is the population[47], D_{ij} is the distance, and m^2 is a square mile.[48]

Considering more than two population centres in space, the total potential will be equal to the sum of the bilateral potentials. Stewart (1948) developed a simplified map where the population is distributed on a flat and continuous surface[49] and the potential at any point produced by the entire population was equal to[50]:

$$V_{at\ any\ point} = \int \frac{1}{D} D_P dS \quad (15)$$

D_P corresponding to the density population in the area dS, and D the distance from that area to any point taken into consideration. This represents the total interaction between a subject at one point and the remaining population of all other areas. Taking into consideration the vector property already mentioned, this can also be expressed at point i as (Philbrick 1973: 42):

$$V_i = k \sum_{j=1}^{n} \frac{P_j}{D_{ij}} \quad (16)$$

where V_i is the population's potential for attraction to centre i, K is a constant, P_i is the population and D_{ij} is the distance.

The astrophysicist showed in several studies how the result of his formula could be used to draw a map of the surface by means of the device of 'contours of equipotential'. The latter reminds of a 'synoptic weather chart',[51]as demonstrated in Figure (1) below produced by Stewart. This is also quite evident from the fact that formula (15) is analogous to and derives from the formulas of gravitational fields (6) in physics.[52] The graphic representation assumes the characteristics of a gravitational or magnetic field with its contours of equipotential.

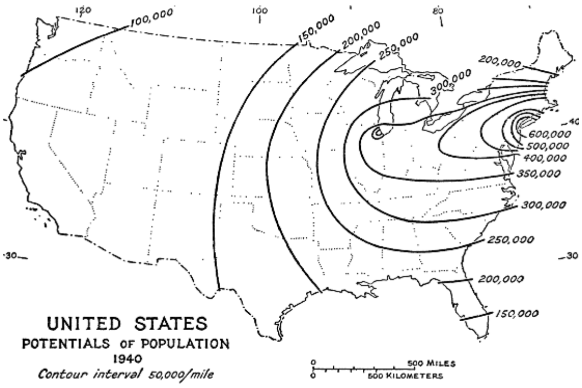

Fig. 1 Contours of equal population potential for the United States, 1940, Source: Stewart (1948)

The potential at any point produced by the entire population can be understood in metaphorical terms as a measure of how close people are to that point. The contribution offered by each person decreases with age.

In addition, the total demographic energy of the whole country can be obtained by combining the formulas (5) and (6) in physics, after several steps it is obtained that:

$$2E_{ij} = V_{ij}P_j + V_{ji}P_i \rightarrow \sum E = \sum P_{state} V_{state} \quad (17)$$

[45] When considering this model, the laws of physics are still valid and they are transposed upon the social phenomenon under analysis. Therefore, any new development or application of socio-physics should comply with such underlying rules.

[46] As seen in physics, in the formula of gravitational potential energy and in that of gravitational potential there is a negative sign because energy is identified with the mechanical work necessary to separate the two particles at an infinite distance, which here is simplified by the scholar as not conceptually significant for the population (Stewart, Ibid., 34-35).

[47] Anderson (1956) considered a stronger distance impact for small centres than for large centres.

[48] Carrothers (1955) suggested that the resistance effect increases with distance but less than proportionally.

[49] Indeed, the method of gravity's representation is generally complex due to its three-dimensional dimension.

[50] Stewart derived this formula from physics: $V = \int \frac{1}{r} DdS$. It considers the potential at any point in a plane and with the presence of multiple masses. In the formula D is the surface density of mass over the infinitesimal element of the area dS, r is the distance from that element to the point under consideration, and the integration is extended to all areas of the plane where D is different from zero.

[51] Also known as 'weather map', it describes the evolving meteorological situation, including pressure and wind, by means of circular lines. Hence the similarities with Stewart's map.

[52] However, it considers the sum of several gravitational potentials through the use of summation.

where P_{state} is the population of a country, and V_{state} is its average potential caused by another country.

Stewart applied these formulas to demonstrate a direct relationship between the demographic energy and other variables, such as the real estate market, the rental market, the value of income, the exchange of letters. This phenomenon acts in space as a kind of gravitational field. For example, starting from the suggestion that rents and the value of land tended to increase as population density increased moving from rural areas to cities, his empirical analysis demonstrates that the demographic energy was proportional to the economic wealth in the US.

Similarly, to Stewart's[53] first formulation, Zipf (1946) described a theory of the movement of people between two communities separated by a certain distance through the formula:

$$F_{ij} = P_i \frac{P_j}{D_{ij}} \quad (18)$$

where F_{ij} is the flow of people between two communities, whose populations are indicated with P_i and P_j respectively, while D_{ij} represents the communities' distance. The scholar also applied his model to the study of the interchange of telephone calls, bus passenger movements, newspaper circulation[54], and other phenomena (Zipf 1949).

In subsequent years, scholars used different weights for the population. By studying New Haven traffic, Rice and Gallagher (1948) adjusted the weights considering the wealth of populations.[55] Furthermore, they introduced the concept of directionally-zoned populations to take into account the "distance to the railroad station" (or to a terminal area), which influenced the traveller's choice of origin city. Stewart (1948) noted that some cities had greater influence, thus assigning different molecular weights to populations of different regions. Dodd (1950) proposed the use of different molecular weights according to sex, age, occupation, income, education and religion. I suggest that these different formulations in Philbrick (1973: 42-43) can be resumed in the formula:

$$F_{ij} = K \left(\frac{(W_i P_i)(W_j P_j)}{D_{ij}^2} \right) \quad (19)$$

where P_i and P_j are the populations (Rice and Gallagher considered the directionally-zoned population of the stations) and W_j and W_i are the weights of the two respective populations (for Rice and Gallagher a wealth factor, for Stewart and Dodd the molecular weights to be estimated).

After a few years, the engineer Alan Voorhees (1956) wrote "A General Theory of Traffic Movement", where he applied gravity to trip distribution.[56] His theory, which represented the first application of the gravitational principle to traffic analysis in urban areas, is used to estimate the number of trips between two zones. According to the scholar, each journey involves four fundamental decisions, often taken simultaneously: namely, the choice of travelling (opportunity cost of distance); the choice of a destination (attractiveness); the choice of means of transport; the choice of route (role of technology and information).

With respect to gravity formulas already seen, the number of trips is inversely proportional to distance, while the size variables considered for the two locations, such as population or level of development, are proportional to the number of trips.[57]

$$T_{ij} = K \frac{P_i P_j}{d_{ij}^x} \quad (20)$$

Drawing on such formulas, Voorhees developed a distribution model, where the estimated number of trips from the area of origin is divided by the destination area. This estimation is proportional to the attraction exerted by each area ('attractiveness of the soil') and is inversely related to the necessary travel time. Its formula is as follows (Philbrick 1973: ibid.):

$$T_{ij} = \frac{O_i \frac{S_j}{d_{ij}^x}}{\sum_{j=1}^{n} \frac{S_j}{d_{ij}^x}} \quad (21)$$

where T_{ij} is the trips between zone i and zone j, O_i is the number of trips generated in zone i, S_j is the attraction force of land j, d_{ij}^x is the travel time between i and j, x is an exponent determined by

[53] Philbrick (1973: 42) points out that although Stewart's and Zipf's formulae are basically the same, Zipf's relationship differs because it raises the entire $\frac{P_i P_j}{D_{ij}}$ to a power.

[54] He analyzes several newspapers including the Chicago Tribune and The New York Times.

[55] Rice and Gallagher are the first to apply an adjustment by a wealth factor to the population, for example by making use of per capita income.

[56] Voorhees was working at the time to predict traffic patterns in Baltimore.

[57] Recent studies reject the idea that size variables (masses) are determined exogenously to the system: economic and demographic totals are usually exogenously given and not unknown variables which need to be determined. These models include attractiveness, which is sensitive to the transport network (Philbrick 1973: 45).

observation,[58] n is the total number of trips including those directed to j.[59]

Eventually, Hansen (1959) was inspired by Stewart and Reilly's ideas when defining the accessibility index (AI)[60], a spatial method for estimating the development of residential land in an urban area. The aggregate growth for a region was calculated by weighted proportionality. In formal terms:

$$AI_i = \sum_{j=1}^{n} \frac{S_j}{D_{ij}^x} \quad (22)$$

$$G_i = G_T \frac{V_i\, F(AI_i)}{\sum V_i\, F(AI_i)}$$

where AI_i is the accessibility index for zone i, S_j is a measure of the activity's size located at zone j (for instance number of jobs), D_{ij} is the separation between i and j with D_{ii} having a numerical value, x is an exponent expressing the effect of separation, G_i is the residential growth in zone i, G_T is the total regional residential growth, V_i is the vacant developable land in zone i, $F(AI_i)$ is a function of the accessibility of zone i.

2.4 Applications of Gravity to Economic Flows

The application of gravity in the study of migration[61], demography and trips comes along with its application to the study of international trade.[62] The economic model is directly inspired by Stewart's model and reproduces its logic and dynamics, but it transposes gravity to a flow of commerce and no longer to a flow of people. The application to the study of international trade traces back to the work of Walter Isard (1954), professor of regional sciences at the MIT University in the United States, but it became known in Tinbergen's (1962) standard bilateral form, which was directly related to Newton's Law of Universal Gravitation. Subsequently, it was re-explored by several economists. Isard was inspired by Stewart's model and introduced the concept of income potential in international trade as in formula (6). Thus, the author applies a gravitational field equation in physics to the study of international trade. In his hypothesis, geographical proximity favours trade due to several variables,[63] such as low transportation costs, cultural and institutional similarities between regions, etc. He stated that a gravity relationship should arise in any model that studies distance as a factor contributing directly to increased costs. Moreover, states' trade is proportional to their respective economic size, which is measured[64] through their Gross Domestic Products (GDPs) or Gross National Products (GNPs). The larger the GDP, the larger the national economy, the more a national population can import or export. Its formula in international economics is as follows (Isard 1954: 308):

$$ {}_i^1V = \sum_{j=1}^{n} {}_i^1V_j = \sum_{j=1}^{n} k \frac{Y_j}{d_{ij}^a} \quad (23)$$

where ${}_i^1V$ is the income potential produced by all countries upon state i and ${}_i^1V_j$ is the income potential produced by state j upon state i, Y_j is the income of state or region j, d_{ij} is the average effective distance between states i and j,[65] k is a constant comparable to the gravitational constant, a is an exponent determined by observation, which is lower than the squared exponent by Stewart.[66]

Later, Tinbergen (1962) developed the gravity equation commonly applied to subsequent studies of international economics, which was intuitive and derived directly from Newton's equation (1). The author was a pioneer of the empirical application of gravity to trade in his work "Shaping the World Economy". In this way, he created a simple bilateral

[58] X indicates the distance resistance rate modelled on the number of trips when the origin and destination are the furthest apart. As the exponent increases, the level of attractiveness of the most distant destinations from the origin decreases. The ratio between S_j/d_{ij}^x represents the perceived utility of a person located at point i with respect to the destination at point j.

[59] The division by the summation that considers the total number of trips (n). It simply divides the utility of j by the sum of the utility of all destinations including j (Voorhess 1956).

[60] Hansen (1959) was the first to introduce the concept of accessibility as a potential of opportunities for interaction. This index, conceptually linked to Newton's law of gravity and by definition to localization theories, has been a great success in urban geography and spatial modelling over the last 60 years, as it considers both the type of activity of people and the connections of the means of transport.

[61] More recently, Anderson (2011) has developed a gravity model for migration through discrete choice techniques and Ahlfeldt et al. (2012), drawing on the contribution of Eaton and Kortum (2002), have built model of commuting gravity.

[62] Among the applications of gravity to the social sciences, the field of trade has been the most fortunate. The model was considered by Deardorff (1998) as "the workhorse for International Economics studies" for its predictive power in estimating exchange flows (Gopinath et al. 2014)

[63] The income potential varies inversely to the distance. Between a group of states, the one more similar in culture and technology to nation i, will have a greater income potential on i.

[64] In 1954 Harris (1954) and Isard and Freutel generalized the concept of mass in the gravitational model with other variables and used transport costs rather than distance costs.

[65] Note that distance as applied in trade is part of the 'resistance factor' which includes all the variables acting as trade barriers, as a kind of 'wedge' tax imposing costs and influencing choices. Moreover, according to Isard, distance should cover the relative movement of goods on diverse means of transport.

[66] The attentive reader will have noticed that, compared to Stewart, Isard loses the squared exponent. Therefore, although Isard's model also follows sociophysics, it does so to a lesser extent than the astrophysicist.

model for the analysis[67] of trade between pairs of countries.[68] The formula is the following:

$$F_{ij} = A \frac{Y_i Y_J}{D_{ij}} \qquad (24)$$

where F_{ij} is the value of trade between country i and country j, A is a constant comparable to the gravitational constant, Y_i is the GDP of country i, Y_J is the GDP of country j, D_{ij} is the distance between the two countries. [69]

Gravity models have also been used similarly and extensively in economics to evaluate FDI flows. Building on the model by Tinbergen (1962), which is suitable for all types of international trade flows, the pioneers of the application of gravity to such cross-border transactions seem to be Eaton and Tamura (1994), Graham (1994) and Frankel et al. (1996).[70] Indeed, governments increasingly perceive FDI as a driver for development[71]. Therefore, it plays a pivotal role in development policies and in negotiations on international trade agreements. Intuitively, the membership in trade agreements, which are set to decrease barriers, should help to increase investments. In addition, the existence of an official language, a common currency or a common legal system among countries should increase the opportunities for investment. In the case of FDIs, the formulation of the model is the following:

$$FDI_{ij} = A \frac{y_i y_j}{D_{ij}} \qquad (25)$$

with FDI_{ij} representing the FDI flow between two countries i and j; A is a constant; y_i is the GDP or GNP of country i; y_j is GDP or GNP of country j; and, finally, D_{ij} is the distance between countries i and j.

Afterwards, the economist[72] and geographer David Huff (1963) re-applied[73] gravity to consumers' behaviour with the aim of forecasting market share and retail attraction. In a set of alternatives, the attractiveness of each retail for a subject is inversely proportional to the perceived utility of any other alternative retail. For example, among competing stores, space is conceptualized as the area of probability within which a customer could decide to shop at a given store. This probability is calculated through the use of distance and attractiveness variables and taking into account the competition. In Huff's standard model there is no differentiation of goods, no feedback between consumers, no alternative goods. The attractiveness of the retail outlet at location j on consumer demand at location i is defined as:

$$\frac{attractiveness\ j = size\ of\ retail\ outlet\ j}{D_{ij}^x} \qquad (26)$$

The probability that this consumer demand[74] at location i is satisfied by the store j will be equal, as already seen in Reilly's equation (11), to the relative attractiveness of j compared to the attractiveness of other stores. As in the case of Voorhees's model, it is worth recalling that x is an exponent expressing the effect of distance decay. Ergo, total demand is distributed by multiplying expression (25). Huff therefore developed a distribution model of consumer demand in space drawing on Voorhees' model, as from formula (19). Similar to the latter, who chooses a spatial distribution of trips based on the attractiveness of the soil, Huff also considers a spatial distribution, which is related to the attractiveness of the commercial area. Huff's probability model is as follows:

$$P_{ij} = \frac{\frac{S_j}{d_{ij}^x}}{\sum_{j=1}^{n} \frac{S_j}{d_{ij}^x}} \qquad (27)$$

where P_{ij} is the probability that a consumer at point i goes to the store located in j, S_j is a measure of store attractiveness (such as store size), d_{ij}^x is the travel time between i and j, x is an exponent

67 While Isard was interested in a complete theoretical approach and an all-inclusive analysis between economic geography and international trade, Tinbergen was more concerned about the empirical aspects.

68 The model predicts bilateral trade between pairs of states.

69 Tinbergen clarified that the economic size of the importing country plays a dual role, indicating both total demand and a sort of production diversity's degree. Tinbergen empirically found, however, that richer countries traded "less than normal", if we were to consider a linear relationship.

70 Other relevant studies were concluded by Frankel and Wei (1996), Brenton et al. (1999), Portes and Rey (2005), Helpman et al. (2008) and by Bergamo and Pizzi (2014). Further uses of the gravitational model in economics were those of Portes et al. (2001), Martin and Rey (2004), Portes and Rey (2005), who build a ravitational model of portfolio investment, and Okawa and van Wincoop (2012) who adapt the gravitational model to the world of international finance.

71 The increase in foreign direct investment flows began with globalisation and has continued over the years, but it was interrupted by a fall in flows due to the international financial crisis (during 2008).

72 David Huff, professor of Marketing and Geography, proposed a model that bridges geography and business. This model became popular over the years for its ease of use and its applicability to a wide range of problems. It has been extensively used by market analysts to locate stores, shopping malls, standard models for the industry and other types of retail establishments.

73 As, to some extent, already seen in Reilly (1929).

74 The demand is inelastic, that is, if there is only one store, it will be totally monopolized by it.

determined by observation, *n* is the total number of stores that also includes the store *j*. The larger the size of a store, the greater the level of attraction exerted on the consumer. The probability that customers choose that store, instead, decreases as distance increases.

After that, the economist Quandt (1965) created[75] a gravitational model of travel[76], namely the gravity demand model, linked to its socioeconomic variables with respect to the areas of origin and destination. According to the following formula (Philbrick 1973:46-47)[77]:

$$T_{ij} = GP_i^{\alpha_1}P_j^{\alpha_2}Y_i^{\alpha_3}Y_j^{\alpha_4}M_i^{\alpha_5}M_j^{\alpha_6}W_i^{\alpha_7}W_j^{\alpha_8}H_{ij}^{\alpha_9}C_{ij}^{\alpha_{10}}S_{ij}^{\alpha_{11}} \quad (28)$$

where T_{ij} is the trips from origin *i* to destination *j* by all modes, P_i and P_j are the populations of the two zones, Y_i and Y_j are their mean personal incomes, M_i and M_j are the population of the labour force employed in mining and manufacturing, $W_i$and W_j are the population of the labour force employed in white collar jobs, H_{ij} is the average travel time, C_{ij} is the average cost of travelling, S_{ij} is an index denoting the number of means of transportation available, G and α are parameters to be estimated.

2.5 Tobler's Law

A few years after the first applications of the Universal Gravitation Law to social sciences, Waldo Tobler (1970[78]), professor of Geography at the University of Michigan, formulated a general law for space science, which had a remarkable affinity with Newton's law and was named 'the first law of geography'. Indeed, Tobler was inspired by gravity models when formulating his theory. The author aimed at creating a more general spatial law through distance in geography, in such a way as to include both spatial interaction phenomena, among which gravitational models and more static phenomena.

This law, then used in social sciences, states that "everything is related to everything else, but near things are more related than distant things" (Tobler 1970). This mechanism, as already illustrated, is known as 'distance decay'. Accordingly, his theory was based simply on predictive spatial power by focusing on the general effect of distance friction, while leaving aside the cause of distribution. Moreover, it focused on '*everything*' instead of on measurable[79] masses as in gravity, leaving more space for the study of non-material concepts such as knowledge, culture, ideas, et cetera. Although it is more general, Tobler's law loses on the side of spatial interaction compared to standard gratisvational models. Indeed, the latter not only provided a static illustration of real distance decay phenomena in social science but were also rather dynamic. Gravity, as already observed, is inclusive of attraction logics and balance of forces between the various subjects considered. These modify the surrounding reality which cannot therefore be static.

The first law of geography, just like the gravity model[80], has been used over time to describe a series of spatial human interaction models (Rey 2015):

$$T_{ij} = \frac{P_i P_j}{D_{ij}^{\beta}} \quad (29)$$

where T_{ij} is the degree of spatial interaction (migration, trade flows, air travel, et cetera) between place *i* and *j*. T_{ij} is proportionate to the sizes of the population and inversely correlated to distance. P_i is the population at the origin and P_j at the destination.[81] D_{ij} is the distance between these locations.[82] The strength of decline in interaction with increasing separation is estimated by β.

[75] Later, it will be developed by the economist himself as well as by his colleagues and students. The interested reader is addressed to: Richard E. Quandt, "The collected essays of Richard E. Quandt", Edward Elgar Publishing 363(1992): 1-864.

[76] The work done by Quandt has been related to intercity travel (dynamic forecasting models) through several reformulations and adaptations of the original model.

[77] The keen reader will have noticed the following formula is similar to the general classical equation to the Cobb–Douglas production function.

[78] However, it is only since the 1990s that Tobler's Law has been rediscovered in literature and used in geographic and social research. (Daniel Z. Sui 2004).

[79] Today it is possible to describe the first law of geography through spatial autocorrelation or Moran's Index (1950). It is defined as a set of unit similar parameters in space. If they are spatially concentrated, there will be a positive spatial autocorrelation. Otherwise, there will be a negative spatial autocorrelation (or spatial heterogeneity). For example, the first law of geography was a conceptual basis of geostatistics, the branch that deals with evaluating spatial autocorrelation of data and checking whether observations made around nearby points have a greater correlation.

[80] In literature, often gravity model and first law of geography are considered a single model.

[81] In this formula, population is used as a variable for size. However, as mentioned, Tobler's model was general and applicable to several variables, such as telephone numbers, income, trade flow, marriage, etc.

[82] Tobler generally used Euclidean distance, but he was open to different variables. Therefore, distance is estimated in the literature using various geographical and commercial policy variables, such as bilateral distances, tariffs (dollars), the presence of the same or different currencies, the presence of regional agreements (RTAs), etc.

Tobler (1970) also tried to identify not only the flow of a spatial interaction but also the distances and positions of the locations, with flows as data values and distance as an unknown variable. Therefore, he was reversing his law by stating that "all things can be located on a map so that similar things are placed closer together than dissimilar things" (Waters 2018:4), thereby creating a deterministic gravitational model of both flows and distances. Thus, the previous formula turns into:

$$D_{ij} = \left(\frac{P_i P_j}{T_{ij}}\right)^{1/\beta} \tag{30}$$

In Tobler (1970), the geographer for the first time applied the reversal of gravity model to derive empirically the position and the diffusion of botanic species between the islands of New Zealand. He demonstrated that distance[83] and size alone can explain up to one-third[84] of the floristic biodiversity between the islands.

In the same vein, Tobler and Wineburg (1971:39-41) estimated the unknown positions of a series of Assyrian settlements between 1940 BC and 1740 BC for which commercial data were available.[85] Using the gravity model to calculate distances, mapping became easier with increasing estimates of distance pairs.[86] In their hypothesis, the more settlements traded with each other, the more similar they were in terms of location. They also assumed that the name occurrences of larger towns were more frequent with respect to smaller ones. [87] Overall, it is both the size of the cities and the distance between them that determine their interactions.

With respect to the applications deriving directly from gravity, Tobler's law, although strictly related to the former, adopts a more general approach focused on geometry. If compared with the eco-physical approach, which proposes the notions of attraction, magnetism and economic relationships between agents, Tobler's model is biased towards geography, distance decay and phenomena of concentration and dispersion.

2.6 Most Recent Applications

Since 2000 a series of spatial interaction's gravity models has been developed under the name of "two-step floating catchment area" (2SFCA) for the accessibility to some public and private services mainly related to health, such as those of GPs, cancer treatment facilities, access to healthy food retailers, et cetera. This method based on spatial decomposition was advanced by Radke and Mu (2000), who combined several related information in a single significant index that allowed for comparisons between services accessible in different locations. More specifically, these models can be divided into five research categories. While the first involves the original dichotomy form of distance decay, the second includes discrete multilevel or continuous shapes also considering density. The third category refers to the development of river basins and the fourth takes into account the competition between supply and demand. Lastly, the fifth considers the means of transport, which integrate into the model, for instance, commuter transport.[88]

The gravity model has also been applied to studies on digital networks. Firstly, and from a theoretical perspective, Batty (1997) introduced the concept of virtual geography, while Gorman and Malecki (2000) coined the term 'cyber-geography' to describe the Internet as a non-homogeneous system with different levels of diffusion across geographical areas. The gravitational metaphor has moved from the study of physical to virtual space. These models studied mutual relationships of the two areas, namely the effect of digitalization on physical distance (Reggiani et al. 2010; Tranos 2011), and the impact of the latter on digital space (Barabási et al. 2012; Newman 2003 and Watts 2004). The most recent applications of digitization by Tranos and Nijkamp (2013) highlighted that the spread of the internet is in turn conditioned by physical, technological, and institutional distance, following a model similar to Newton's Gravitational model.

Anderson et al. (2015) introduced a structural gravity model to analyse international trade in different services, sectors, countries and time. The authors paid greater attention not only to trade distances, but also to variables that are important for services, such as institutions, digital infrastructures, et cetera thereby creating a new methodology for deriving services' border barriers, which by their nature are difficult to obtain. According to the scholars, in fact, little is known about the cost of trade in services, a sector that is characterized by the lack of data. This is due to the absence of transport

[83] Despite using a geometric distance, Tobler was aware that this was a simplification of reality.

[84] Tobler did not believe that his model could explain all variations, but he thought that more sophisticated models could later be developed on this simpler basis. In that regard, it stated that: "Model-building is useful not only because it may allow prediction but also because it identifies areas for further research" (Claude Grasland 2010: 4).

[85] On the archaeological data found by Hrozny near the village of Kültepe in 1925.

[86] Tobler located the cities through a process of comparison between estimates of the inverse of the law of gravity and archaeological sites discovered in that region.

[87] The size of the cities was not known, neither was the value of the flows. Tobler estimated the size of cities from the frequency of quotations and flows from the common citation of names.

[88] The interested reader will be able to deepen in Luo and Wang (2003) and 2005, Luo and Qi (2009).

costs, opaque laws and difficult-to-measure trade's tariffs in professional and financial services compared to trade flows of goods. Anderson et al. (2015) concluded that the gravitation model fits well with the service sector. Over time, border barriers have generally decreased for larger countries with differences by sector, while staying fairly stable for smaller economies. In addition, the negative distance effect seems to be stronger for neighbouring areas and it is reduced more than proportionally over long distances. On the contrary, language effects are more influential for services trade than for goods trade.

More recently, Bonchek (2020) suggested the application of gravity to business economics and marketing. Indeed, once companies have built their brand identity, they produce a gravitational field that attracts customers into the orbit of their brand. [89] In the most famous companies, this field of attraction would be created through three steps: shared purpose, engagement platforms and collaborative partners. While in the first step the attention is not on the immediate purchase, but on sharing a purpose to attract future consumers, the second addresses engagement platforms keeping the customers in the orbit around the brand. The last step consists of creating partners that amplify the gravitational field of the brand by multiplying the value of services and strengthening corporate credibility. By following these phases, the company builds relationships with its consumer base through authenticity and social connections.

3. CONCLUSIONS

The socio-physical importance of the gravity model has been widely discussed, starting with the birth of the model in physics and its transposition to several domains, especially those related to human behaviours and social phenomena. To that end, attention was paid to the exposition of Newton's law and to the preservation of the physics approach as a method of analysis, while also introducing the concept of 'social-physics'. A great deal of consideration was given to the most common applications of the gravitational model, as for economic, migratory, and demographic fields. The review followed a chronological order of presentation that favours a better understanding of the development of the use of gravity in the social sciences over time. Furthermore, a division by topic was outlined.

In particular, considering all the models scrutinized, two large lines of study could be distinguished (with the exceptions of Pownall's model): Tobler's first law of geography, and the most recent models of gravity. Although they are linked to other models by the common application of gravity to social sciences, these lines of thought developed more independent theories. Following a chronological order, the first study focused on the flow of people declined in its specific models such as migration, demography attraction, trips, et cetera. Drawing on the latter, the second one studies economic flows (trade, investment, etc.), thereby shifting from the investigation of people's movement to the movement of things. The link between these two fields of study can be historically found in the seminal works by Stewart and Isard. Indeed, through the wide extension of physics to his social model, Stewart laid down the theoretical foundations for subsequent models. Isard, who was inspired by Stewart's demographic approach, developed his model of international trade. In light of the most recent models reviewed and partly due to the more general nature Tobler's first law of geography, a new line of study seems to emerge. The latter moves from the analysis of material things (either individuals, money or goods) to less tangible concepts, such as the offer of services, brand identity, et cetera.

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